

Principles of Nuclear reactors

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Vietnam Atomic Energy Commission

This lecture consists of two parts:

1 . Nuclear physics and reactor principles (1-52)

by NGO QUANG HUY , Hochiminh city

2 . Introduction to thermal hydraulics (53-98)

by CAO CHI ,Hanoi

Part I

NUCLEAR PHYSICS AND REACTOR PRINCIPLES

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NUCLEAR PHYSICS AND REACTOR PRINCIPLES

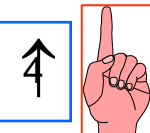
I. Nuclear Physics

II. Interaction of neutron with material

III. Critical state of nuclear reactor

IV. The time-dependent reactor

V. The operational reactivity effects



I. NUCLEAR PHYSICS

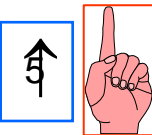
1.1. Atomic and nuclear structure

An *atom* consists of a small but massive positive *nucleus* surrounded by a cloud of rapidly moving negative *electrons*.

The nucleus is composed of *protons* and *neutrons*, common name of which is *nucleon*.

The total number of protons in the nucleus is called the *atomic number* of the atom and is given the symbol Z .

The number of neutrons is denoted by N . The total number of nucleons in nucleus is $A = Z + N$, which is called the *atomic mass number*.



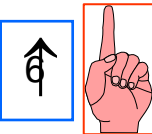
Atoms whose nuclei contain the same number of protons but different number of neutrons are known as *isotopes*.

For example, oxygen has three *stable isotopes* ^{16}O , ^{17}O , ^{18}O and five unstable or *radioactive isotopes* ^{13}O , ^{14}O , ^{15}O , ^{19}O and ^{20}O .

The atomic mass unit, abbreviated amu, is defined as one-twelfth the mass of neutral ^{12}C atom, that is:

$$1 \text{ amu} = \frac{1}{12} m(^{12}\text{C}) = 1.66053 \cdot 10^{-27} \text{ kg}$$

(1.1)



1.2. Atomic and nuclear radii

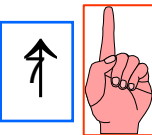
The average radii of atoms, except for a few of the lightest atoms, are about $2 \cdot 10^{-10}$ m. Nuclei can be considered as sphere with a radius given by the following formula:

$$R = r_0 \cdot A^{1/3} \quad (1.2)$$

$$r_0 = (1.2 - 1.5) \cdot 10^{-15} \text{ m}$$

From the formula (1.2), it follows that the volume V of nucleus is proportional to A . So the ratio A/V , the number of nucleons per unit volume is a constant. The density of nuclear material is:

$$\rho = \frac{m}{V} = \frac{Am_p}{\frac{4}{3}\pi r_0^3 A} = \frac{m_p}{\frac{4}{3}\pi r_0^3} \approx 10^{14} \text{ kg/m}^3$$



1.3. Mass and energy

The Einstein's formula:

$$E = mc^2 \quad (1.3)$$

Where $c = 3 \cdot 10^8$ m/s is the speed of light.

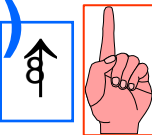
Unit of energy in SI system is Joule (J). Another unit often used in nuclear engineering is *electron-volt*, denoted by eV.

$$1 \text{ eV} = 1.60219 \cdot 10^{-19} \text{ J} \quad (1.4)$$

The relationship between the atomic mass unit and eV is:

$$1 \text{ amu} = 931.481 \text{ MeV} \quad (1.5)$$

where $1 \text{ MeV} = 10^6 \text{ eV}$.



When a body is in motion, its mass increases relative to an observer according to the formula:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad (1.6)$$

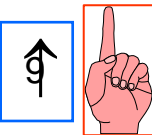
where m_0 is the rest mass and v is its speed.

The kinetic energy E is the difference between the total energy W and the rest-mass energy:

$$E = mc^2 - m_0c^2 = \frac{m_0}{\sqrt{1 - v^2/c^2}} - m_0c^2 \quad (1.7)$$

In the non-relative case, i.e. when $v \ll c$, the kinetic energy becomes:

$$E = mv^2 \quad (1.8)$$



1.4. Binding energy

Mass defect:

$$\Delta M = Zm_p + Nm_n - M(Z,A) \quad (1.9)$$

where m_p and m_n are the masses of proton and neutron respectively, and $M(Z,A)$ is the mass of the nucleus concerned.

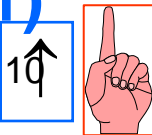
The energy equivalent of the mass defect is called the **binding energy** E_B of the nucleus:

$$E_B = [Zm_p + Nm_n - M(Z,A)]c^2 \quad (1.10)$$

Average binding energy per nucleon versus A (Fig. 1.1):

$$\varepsilon = \frac{E_B}{A}$$

(1.11)



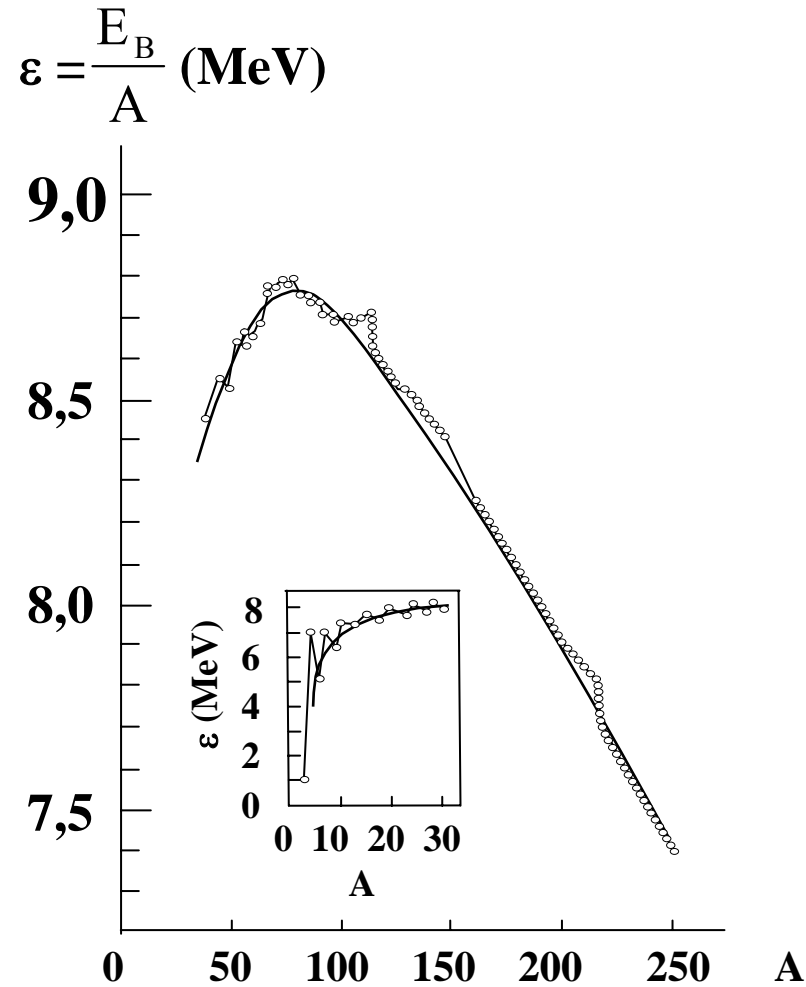
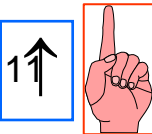


Fig. 1.1. Binding energy per nucleon as function of atomic mass number.



From Fig. 1.1 it is noted that the curve ε increases with A increasing from 1 to about 50 and after that it is a decreasing function of A .

This behaviour of the binding energy curve is important in determining possible sources of nuclear energy. Those nuclei in which the binding energy per nucleon is high are especially stable or tightly bound.

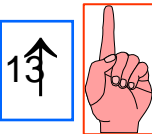
Fig. 1.1 shows that the middle nuclei are the most stable, when the lighter or heavier nuclei are less stable. Binding energy can be released either *from light nuclei by fusion* or *from heavy nuclei by fission*.



II. INTERACTION OF NEUTRON WITH MATERIAL

2.1. The types of interaction of neutrons with nuclei

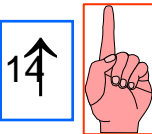
Since neutrons are electrically neutral, they are not affected by the electrons in an atom or by the positive charge of the nucleus. Therefore the neutrons pass through the atomic electron cloud and interact directly with the nucleus. That means, neutrons collide with nuclei but not with atoms. Neutrons interact with nuclei in one or more of the following ways:



a. Elastic scattering: In the elastic scattering the neutron strikes the nucleus, the neutron reappears and the nucleus is left in its ground state. This type of scattering is abbreviated by the symbol (n,n) . The elastic scattering is the main mechanism in moderating neutrons in thermal nuclear reactors.

b. Inelastic scattering: This process is identical to elastic scattering except that the nucleus is left in an excited state. The excited nucleus decays by the emission of γ rays. Inelastic scattering is denoted by the symbol (n,n') .

c. Radiative capture: Here the neutron is captured by the nucleus, and one or more γ rays are emitted. Since the original neutron is absorbed, this process is an example of a class of absorption reactions. The radiative capture is denoted by the symbol (n, γ) .



d. Charged-particle reactions: Neutrons may also disappear as the result of absorption reactions of (n,α) and (n,p) .

e. Neutron-producing reactions: Reactions of the type $(n,2n)$ and $(n,3n)$ occur with energetic neutrons. The $(n,2n)$ reaction is important in reactors containing heavy water or beryllium because ^2H and ^9Be have loosely bound neutrons which can be easily be ejected.

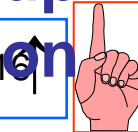
f. Fission: Neutrons colliding with certain nuclei may cause the nuclei to split apart to undergo fission. This reaction is the principal source of nuclear energy.



2.3. Fission reaction

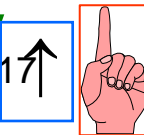
In fission reaction, a heavy nucleus, for example uranium, is subjected to neutron bombardment and breaks into fragments, such as ^{139}Ba and ^{97}Kr , releasing an energy of approximately 200 MeV. This process is also accompanied by a release of neutrons. There are two types of fission: spontaneous and induced.

a. Spontaneous fission: Heavy nuclei sometimes experience spontaneous fission with no apparent external stimulus. Examples are ^{235}U and ^{238}U , whose respective half-life periods for fission are 1.2×10^{17} years and 5.5×10^{15} years respectively. The fission produced in these cases is insignificant for energy production. However the phenomenon is important since represents an uncontrollable source of neutrons in a reactor and it is, furthermore, possible to make use of it in the start-up stage. An example of the use of this fission is the neutron source of ^{252}Cf .



b. Induced fission: Certain heavy nuclei can be induced to fission, as result of one neutron capture. Consequently, several high-energy neutrons are produced, which permit to maintain the chain reaction process. The nuclei ^{235}U , ^{233}U , ^{239}Pu and ^{241}Pu experience fission with low-energy thermal neutrons and they are called *fissile materials*. While the nuclei ^{238}U and ^{232}Th fission with fast neutrons. The radiative capture of neutrons by ^{238}U and ^{232}Th leads to the formation of the fissionable materials ^{239}Pu and ^{233}U , so they are called *fertile materials*.

^{235}U is found in natural uranium in a proportion of 0.72%, while the other 99.28% is ^{238}U . The nucleus ^{233}U does not exist in natural uranium and possesses the most desirable properties of all fissile nuclei. During the period of operation of reactor, ^{239}Pu , ^{241}Pu and ^{233}U are formed from the radiative capture of neutrons by ^{238}U and ^{232}Th .



c. Fission fragments: The study of ^{235}U fission with thermal neutrons has demonstrated that the compound nucleus fissions in more than 40 different ways, providing for about 80 fission fragments. Fig. 2.1 demonstrates the relative frequency with which the nuclear fragments are produced for nuclides of a specific mass number. In the fission yield curve, two fragments are produced per fission, so the area below the curve should add up to 200%. It is probable to have mass number of 80 and 160, while the most probable values are 95 and 140. Symmetric fission fragments are very rare.



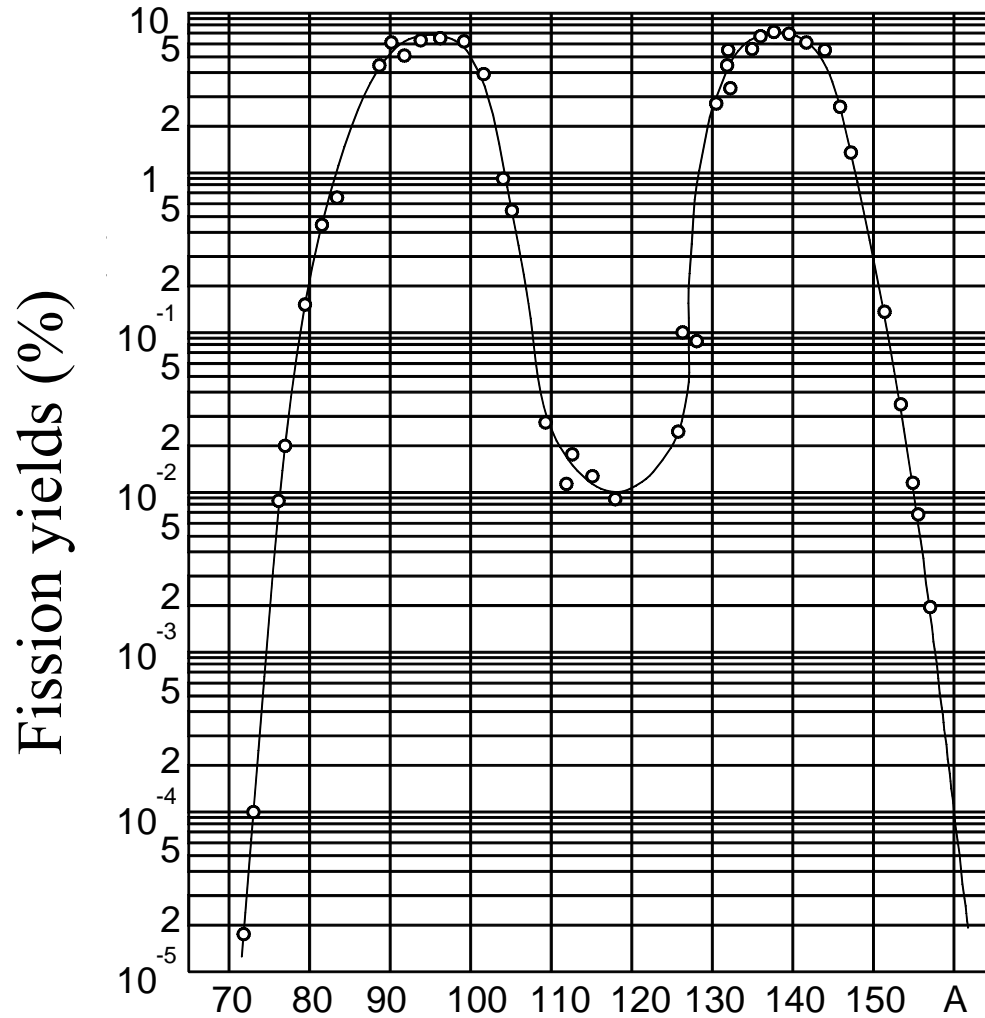


Fig. 2.1. Fission product yields for thermal fission in ^{235}U .

d. Fission products: The two important products of fission reactions are nuclides and particles produced and energy released.

* The nuclei produced in reaction are called fission fragments. Due to the high neutron/proton ratio of fission fragments in relation to stability, they are usually radioactive. They will seek to reduce their n/p ratio by means of successive decay (β^- , γ), until they reach stability. As a result of fission there are approximately 300 known nuclides. The half-life for fission fragments can vary from fractions of seconds to thousands of years. An amount of neutrons, gamma, and beta particles is also produced in fission reactions.

* An energy of approximately 200 MeV is released when a nucleus fissions. This value depends on the fissile nucleus and the fission fragments. For ^{235}U , the approximate distribution of this energy is shown in Table 2.2.

Besides, the contribution of energy of gamma ray of about 8 MeV in radiative reaction (n,γ) leads to the total of 211 MeV. From the point of view of the reactor, we are interested in recoverable energy, and thus the energy released by neutrons is considered lost. So the released energy is approximately 200 MeV.

e. Chain reaction: Two basic requirements, which must be filled in order to produce power in a reactor, are that the fission rate should be high and this rate must be continuously maintained. The fission process may be repeated due to the fact that each fission releases neutrons. The condition for the maintaining of a chain reaction is that the neutron produced in a fission be capable of producing another fission.

For simplification we consider that 3 neutrons are produced in a fission (in practice 2.43 neutrons are produced in a thermal fission of ^{235}U). Due to the fission by these three neutrons we have $3^2 = 9$ neutrons in secondary generation. This chain process continues and is illustrated in Fig. 2.2.

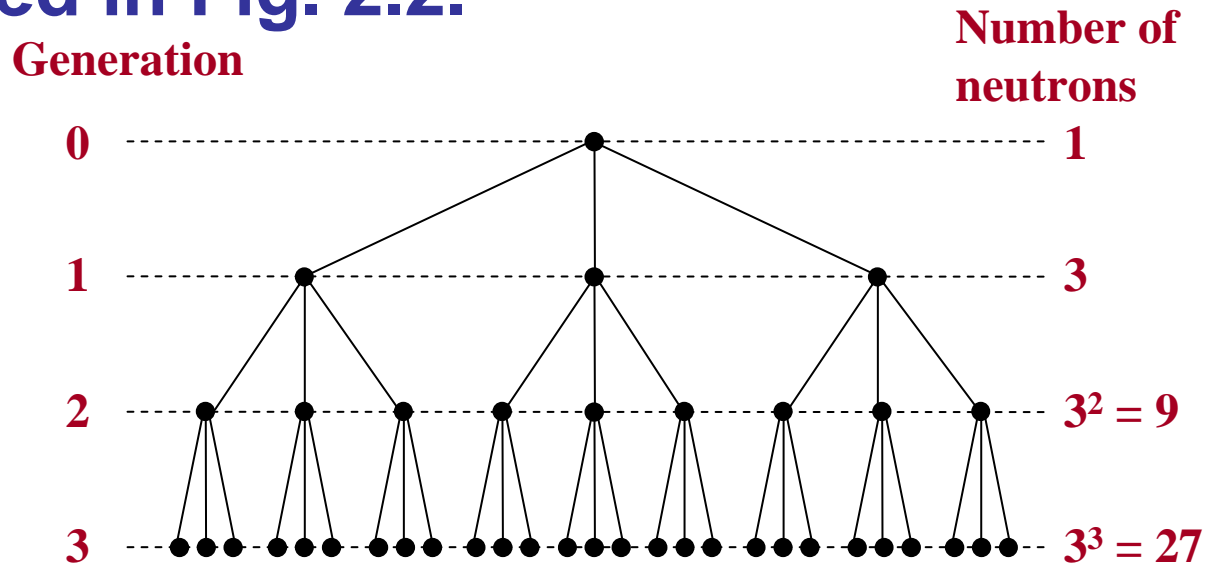
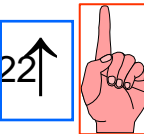


Fig. 2.2. The chain fission process.



2.4. The neutrons of fission reaction

There are two main types of neutrons due to fission reactions: Prompt neutrons and delayed neutrons.

a. Prompt neutrons: The time elapsed between the absorption of a neutron and the production of the fission neutrons is 10^{-14} seconds, or in other words, practically zero. For this reason, these neutrons are called *prompt neutrons*. The number of neutrons emitted in the thermal fission of ^{235}U varies between 0 and 5 and the average number is $\nu = 2.43$. For ^{239}Pu and ^{241}Pu these figures are $\nu = 2.89$ and $\nu = 2.93$ respectively.



b. Delayed neutrons: When fission ceases, the emission of prompt neutrons also automatically ceases, but some fission fragments continue to emit neutrons via radioactive decay. These fragments are called the *precursors* and the neutrons emitted by them are called *delayed neutrons*. The precursors are divided into 6 groups in accordance to their half-life $T_{1/2}$. Table 2.3 shows the characteristics of the delayed neutrons in thermal fission.

It is noted that the proportion β of delayed neutrons is insignificant. For ^{235}U , it is 0.65%, while the proportion of the prompt neutrons is 99.35%. Though the proportion of delayed neutrons is small but they have a very important effect in the control of the reactor.



c. Neutron cross sections: The interaction of the neutrons with matter is described in terms of quantities called ***cross sections***. They are defined as the interaction rate per atom in the target per unit of intensity of a neutron beam. The cross section has the dimension of an area, and usually is measured in units of barns (b), and $1 \text{ b} = 10^{-24} \text{ cm}^2$.

2.5. Diffusion equation

a. Diffusion equation: The diffusion equation describes the diffusion of neutrons in medium. It is obtained from a balance equation between production, absorption and leakage of neutrons in a given volume:

$$\frac{1}{v} \frac{\partial \Phi}{\partial t} = D \Delta \Phi - \Sigma_a \Phi + S \quad (2.5)$$

where Φ is the neutron flux, D is diffusion coefficient, Δ is Laplace operator, Σ_a is macroscopic absorption cross section, v is neutron velocity and S is neutron source.

In the steady state case the rate of change of neutrons is zero, so the equation (2.5) becomes:

$$D \Delta \Phi - \Sigma_a \Phi + S = 0 \quad (2.6)$$

c. Solutions of one-group diffusion equation: For simplification we consider that the production, diffusion and absorption of neutrons occur at the thermal energy. Then the source of thermal neutrons S is:

$$S = k \Sigma_a \Phi \quad (2.10)$$

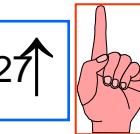
The diffusion equation (2.6) becomes *one-group diffusion equation*:

$$\Delta \Phi + B^2 \Phi = 0 \quad (2.11)$$

where

$$B^2 = \frac{k-1}{L^2} \quad (2.13)$$

$$L^2 = \frac{D}{\Sigma_a} \quad (2.14)$$



III. CRITICAL STATE OF NUCLEAR REACTOR

3.1. Neutron multiplication factor

a. Neutron multiplication factor k

The infinite neutron multiplication factor is the ratio between the numbers of neutrons in one generation to the past generation in an infinite medium:

$$k_{\infty} = \frac{n_2}{n_1} \quad (3.1)$$

- If factor $k_{\infty} = 1$, the reactor is in critical state
- $k_{\infty} < 1$, the reactor is in subcritical state
- $k_{\infty} > 1$, the reactor is in supercritical state

In a critical reactor, one neutron of each fission will cause exactly one more fission. The neutron life cycle is illustrated in Fig. 3.1. In a fission, about 2.43 fast neutrons are born. In critical reactor, only one neutron completes the cycle to maintain the chain reaction. The other 1.43 neutrons are lost via leakage or capture.

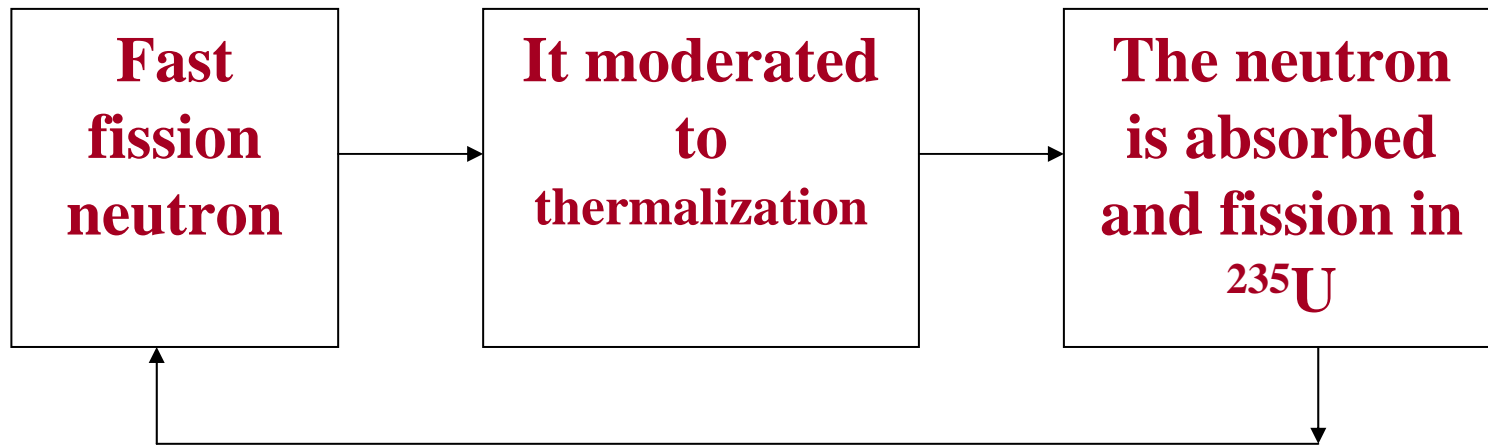


Fig. 3.1. The neutron life cycle.

The *effective neutron multiplication factor* k is applied for a finite medium, i.e. for a reactor with a limited size. In this case, the effective neutron multiplication factor is:

$$k = k_{\infty} P_f P_t \quad (3.2)$$

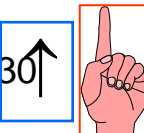
where: P_f is the fast-neutron non-leakage probability during moderation

P_t is the thermal-neutron non-leakage probability during diffusion

$$P_f = \frac{1}{1 + B^2 L_s^2} \quad (3.3)$$

$$P_t = \frac{1}{1 + B^2 L^2} \quad (3.4)$$

L_s is the fast neutron moderation length; L is the thermal diffusion length; B^2 is Buckling.



The formula (3.2) becomes:

$$k = k_{\infty} \frac{1}{1 + B^2 L_s^2} \frac{1}{1 + B^2 L^2} \quad (3.5)$$

b. Four factors formula

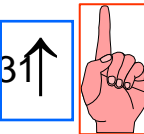
To take in account the neutron life cycle we have the four factors formula for the infinite multiplication factor as follows:

$$k_{\infty} = \eta \mu p f \quad (3.6)$$

The thermal fission factor η : This presents the number of fast neutrons produced for each thermal neutron absorbed by the fuel:

$$\eta = \frac{\text{Fast neutrons produced by thermal neutron}}{\text{Thermal neutrons absorbed in fuel}}$$

$$\eta = \nu \frac{\sigma_{f5} N_5 \Phi}{\sigma_5 N_5 \Phi + \sigma_8 N_8 \Phi} = \nu \frac{\sigma_{f5} N_5}{\sigma_5 N_5 + \sigma_8 N_8} \quad (3.7)$$



The fast fission factor μ : this is the ratio between the fast neutrons from all fissions and the fast neutrons from thermal fission .

$$\mu = \frac{\text{(fast neutrons from all fissions)}}{\text{(fast neutrons from thermal fission)}} \quad (3.8)$$

The resonance escape probability p : This is the measure of how many neutrons can go through resonances without being absorbed:

$$p = \frac{\text{Neutrons that reach thermal energy}}{\text{Fast neutrons that starts to slow down}}$$

$$p = \exp\left(-\frac{N \int_{\xi} I_{\text{eff}}}{\Sigma_s}\right) \quad (3.9)$$

where I_{eff} is the effective resonance integral.



The thermal utilization factor f : This is the fraction of thermal neutrons absorbed by the fuel.

$$\mathbf{f} = \frac{\text{Number of neutrons absorbed by fuel}}{\text{Number of neutrons absorbed in reactor}}$$
$$\mathbf{f} = \frac{\sigma_5 N_5 + \sigma_8 N_8}{\sigma_5 N_5 + \sigma_8 N_8 + \sum_i \sigma_i N_i} = \frac{\sigma_u N_u}{\sigma_u N_u + \sum_i \sigma_i N_i} \quad \mathbf{(3.10)}$$

3.2. Critical size of the reactor

When the size of reactor is such that the chain reaction is maintained, we refer to this as **critical size**. If the size of reactor is less than critical size, the neutron leakage is larger, so the reactor will be at **subcritical state**. If the size of reactor is more than critical size, the neutron leakage is smaller, so the reactor will be at **supercritical state**.

The critical condition of reactor is

$$k = k_{\infty} P_f P_t = \eta \mu p f \frac{1}{1 + B^2 L_s^2} \frac{1}{1 + B^2 L^2} = 1 \quad (3.11)$$

This condition is equivalent to the following condition:

$$B_{\text{g}}^2 = B_{\text{m}}^2 \quad (3.12)$$

where B_{m}^2 and B_{g}^2 are the *material Buckling* and *geometrical Buckling*, respectively.

$$B_{\text{m}}^2 = \frac{k_{\infty} - 1}{M^2} \quad (3.13)$$

where M^2 is the *migration area*.

$$M^2 = L_s^2 + L^2 \quad (3.14)$$

The geometrical Buckling B_{g}^2 has different values depending on the geometry of reactor (see section 2.5c).

If the reactors with different geometries have the same material Buckling, the ratios between their volumes are:

$$V_{\text{Parallelepiped}} : V_{\text{Cylinder}} : V_{\text{Sphere}} = 1,24 : 1,145 : 1$$

(3.25)

So the *spherical reactor has the smallest volume* and the *parallelepiped reactor has the largest volume*. In practice, one often chooses *the reactor having cylindrical shape*.

IV. THE TIME-DEPENDENT REACTOR

4.1. Reactivity

A reactor is critical when the effective multiplication factor k is equal to 1. The factor that determines how subcritical or supercritical a reactor may be is the following quantity:

$$\rho = \frac{k-1}{k} \quad (4.1)$$

This is called *reactivity*. In practice, k is always very close to 1, so:

$$\rho \approx \delta k = k - 1 \quad (4.2)$$

4.2. The effect of the prompt neutrons

Assume that at the beginning moment $t = 0$ the neutron concentration is $N(0)$. The life time of prompt neutrons is λ . The neutron at the moment t is determined by the following formula:

$$N(t) = N(0) e^{t/T} \quad (4.3)$$

$$T = \lambda / \delta k \quad (4.4)$$

where T is called *reactor period*. According to the formula (4.4), when $\delta k > 0$ the neutron concentration N will increase and when $\delta k < 0$ it will decrease.

Let us see what would happen if all of neutrons are prompt. The life time of H₂O moderator is $\approx 5 \cdot 10^{-4}$ s. Assume that $\delta k = 0.0025$, the reactor period is:

$$T = \lambda / \delta k = (5 \cdot 10^{-4} / 25 \cdot 10^{-4}) \text{ sec} = 0.2 \text{ sec}$$

With $t = 1$ s we have:

$$N(1) = N(0) e^5 \approx 150 N(0) \quad (4.6)$$

That is to say, in 1 sec, the neutron flux or the reactor power will increase 150 times. The reactor control would be impossible under these circumstances, since the power changes too fast.

4.2. The effect of the delayed neutrons

In the fission of ^{235}U , about 99.35% of the neutrons produced are prompt neutrons and the remaining 0.65% is delayed neutrons. The life time of the i th component of delayed neutrons is determined by the half-life of its precursor $T_{1/2,i}$:

$$\lambda_i = (1 / 0.693) T_{(1/2)_i}$$

and the average life time of delayed neutrons is:

$$\lambda_{\text{delayed}} = (1 / \beta) \sum \beta_i \lambda_i = 12,4 \text{ s} \quad (4.8)$$

where Σ = sum over i

The average life time of all neutrons, including prompt neutrons and delayed neutrons, is:

$$\text{Average of } \lambda = (1 - \beta) \lambda + \beta \lambda_{\text{delayed}} = 5 \cdot 10^{-4} + 0.0064 \times 12.4 \approx 0.08 \text{ s} \quad (4.9)$$

With the assumption that $\delta k = 0.0025$, the reactor period becomes:

$$T = (\text{average of } \lambda) / \delta k = 0.08 / 0.0025 \text{ s} = 32 \text{ s} \quad (4.10)$$

With $t = 1 \text{ s}$ we have:

$$N(1) / N(0) = e^{1/32} \approx 3,1\% \quad (4.11)$$

This rise in power is much slower than in the case of the prompt neutrons and thus it is possible to implement protection and regulation systems.

The simple treatment of the reactor period and power changes resulting from a given reactivity only provide an approximation, which is not bad for small reactivities (0.005 or less) but is of no use with larger reactivities. The reactivity limit for the effect of delayed neutrons is

$$\rho_{\text{limit}} = \beta = 0.0065 \quad (4.12)$$

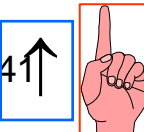
This means that the reactivity of 0.0065 or more places us in a situation of danger since the dynamics of the reactor is governed by prompt neutrons. The unit $\rho/\beta = \$$ is known as “dollar”, and the prompt critical condition is $\$ = 1$.

4.3. The kinetic equations of reactor

Kinetic equations describe the time dependence of neutron concentration N (neutrons/cm³). For simplification we consider the equations *in an approximation of one group of thermal neutrons*, ignoring the neutron moderation. Then we have:

$$\frac{dN}{dt} = \text{neutron production rate}$$

- neutron absorption rate
- rate of producing the fission fragments
- + rate of producing neutrons from fission fragments
- + neutron external sources



This equation leads to the following system of kinetic equations:

$$\frac{dN}{dt} = [k(1 - \beta) - 1] \Sigma_a \Phi + \sum_{i=1}^6 \lambda_i C_i + S \quad (4.13)$$

$$\frac{dC_i}{dt} = \beta_i k \Sigma_a \Phi - \lambda_i C_i \quad (4.14)$$

where C_i is the precursor concentration of i^{th} group of 6 delayed neutrons, S is the external neutron source intensity.

The equations (4.13) and (4.14) do not depend on space coordinates; therefore they are called the *kinetic equations of point reactors*.

V. THE OPERATIONAL REACTIVITY EFFECTS

In practice, there are reactivity changes, which occur when the reactor is operating. This type of reactivity change is caused by the following:

- *Temperature effect:* When reactor power is increased, there is a rise in the temperature of the fuel, the heat transfer fluid and the moderator. These temperature changes affect reactivity in seconds.
- *Poisoning effect:* The accumulation of fission products, which depend on the level of power, absorbs neutrons and reduces reactivity. These changes affect reactivity in several hours.



-Fuel burn-up effect: When fuel burns, its composition changes (the number of atoms in ^{235}U diminishes and ^{239}Pu is produced). As these nuclides have different fission cross sections, reactivity changes are produced as a result of the burn-up process. These changes affect reactivity **in weeks**.

-Void effect

-Doppler effect

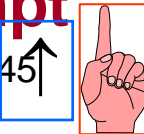
5.1. Temperature effects on reactivity

The extent to which the reactivity is affected by changes in temperature is described in terms of temperature coefficient of reactivity, denoted as α_T . This is defined by the relation:

$$\alpha_T = \frac{d\rho}{dT} \approx \frac{1}{k} \frac{dk}{dT} \quad (5.1)$$

The response of the reactor to a change in temperature depends on algebraic sign of α_T . When $\alpha_T > 0$ the power level increases, and this increases the temperature, which again increases the power level, and so on. In the absence of outside intervention, the power level will increase without limit. When $\alpha_T < 0$, the increase of power level leads to the increase of temperature, and in turn it decreases the power level. Thus, *reactors with positive α_T are unstable and those with negative α_T are stable with respect to changes in temperature.*

In reactors, which operate on solid fuel, the temperature coefficient α_T is of great importance in terms of safety. This is due to the fact that the temperature of the fuel responds almost instantaneously to changes in power, while changes in the temperatures of the moderator and the coolant depend on the heat transfer provided by the fuel. Thus, the fuel α_T is often referred to as the prompt temperature effect.

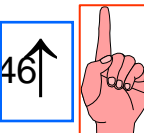


6.2. Xenon poisoning

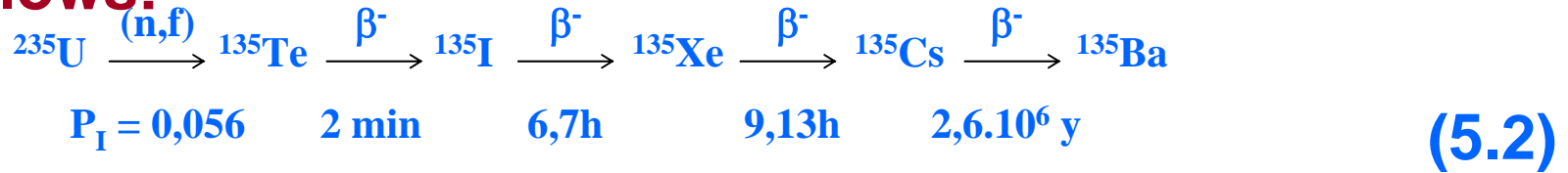
All fission products absorb neutrons to some extent, and their accumulation in a reactor tends to reduce its multiplication factor. Since absorption cross sections decrease rapidly with increasing neutron energy, such fission product poisons are of greatest importance in thermal reactors.

a. ^{135}Xe concentration

The most important fission product poison is ^{135}Xe , whose thermal cross section is 2.65×10^6 barns. This isotope is formed as the result of the decay of ^{135}I and also is produced directly in fission. The ^{135}I is also formed in fission and by the decay of ^{135}Te .



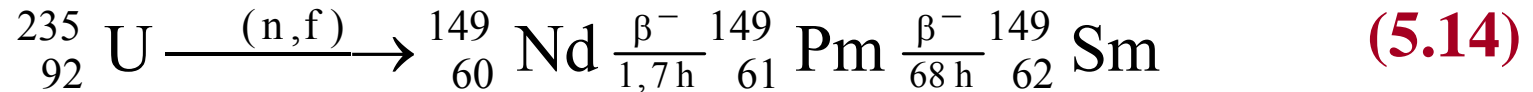
These processes and their half-lives are presented as follows:



where fission product yield of ${}^{135}\text{I}$ is $p_I = 0.056$ and that of direct ${}^{135}\text{Xe}$ is $p_{\text{Xe}} = 0.003$.

5.3. Samarium poisoning

The isotope poisoning the reactor after ^{135}Xe is ^{149}Sm . It is formed by the following process:



^{149}Sm is a stable isotope and ^{149}Nd is a radioactive isotope with the short half-life of 1.7 h.

5.4. Fuel burn-up

Fuel composition changes significantly as a result of reactor operation. In the long-run, this situation changes the ρ , the dynamics of the reactor, the distribution of the neutrons and the need to redistribute/reload fuel into the core. Over the long-run *the fissile atoms of ^{235}U are burned and ^{238}U is changed into fissile ^{239}Pu* . The burn-up rate depends on neutron flux Φ and therefore:

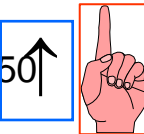
$$\frac{dN_5}{dt} = -\sigma_5 \Phi N_5$$

(5.21)

The higher the Φ the faster the fuel consumption. Fortunately, the ^{235}U consumption is compensated by the formation of ^{239}Pu .

Since ^{239}Pu is produced via the capture of 1 neutron by ^{238}U , and since concentration of ^{238}U does not change significantly during the fuel cycle, we may assume that the ^{239}Pu is formed at a constant rate. Thus, as more and more Pu is formed, the fuel absorbs more and more neutrons; there will come a moment when *the production rate will be equal to the absorption rate.*

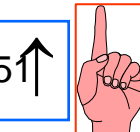
The ^{239}Pu that absorb neutrons may be transformed into ^{240}Pu and when this material absorbs a neutron, it will be transformed into ^{241}Pu . Therefore, following a period of operation, *the fuel will have three different fissile nuclei: ^{235}U , ^{239}Pu and ^{241}Pu .*



The effect of the plutonium production is a function of the initial ^{235}U enrichment. In a core that is enriched to 20% or greater the effect is minimal. In natural uranium core or one only enriched to several percent ^{235}U the effect can be substantial over the life of the core.

Core burn-up is compensated for by initial loading of the fuel to allow for burn-up and the need for excess reactivity. Control rods compensate for this as well as the use of burnable poisons in the fuel meat that will have diminished effect over time.

We may observe how reactivity varies as a result of burn-up. Initially, the burn-up of ^{235}U is replaced by a certain number of atoms of ^{239}Pu , which causes a rise in reactivity.



This is explained by the fact that the σ_f of ^{239}Pu (742 b) is greater than that ^{235}U (582 b). For high levels of ^{235}U burn-up, the burn-up continues, but the accumulation of ^{239}Pu begins to slow down (that is to say, it reaches equilibrium), and as a result, the number of fissile nuclei diminishes and reactivity drops. *When the reactivity reaches about 0.01, fuel replacement is initiated.*



Part II

Introduction to Thermal Hydraulics

Prof. Cao Chi

- I . Why thermal hydraulics ?
- II . Mechanisms of heat transfer
- III . Heat generated and power of the reactor
- IV . Correlations between temperatures
- V . Phases in the coolant
- VI . Pump
- VII. Conclusion

References :

[1] Argonne Basic Courses on Nuclear Safety

[2] Ferdinand Cap ,Physik und Technik der Atomreactoren

[3] John R. Lamarsh ,Introduction to Nuclear Engineering

I . Why thermal hydraulics?

Why we must study thermal hydraulics?

Our purpose is to calculate the temperature at different points in the reactor.

If knowing the temperature we must come back to the physics computation of the reactor , because the temperature affects the cross sections of different interactions (Doppler effect).

But the temperature is on its turn depends on the neutron flux Φ , so we have a mutual feedback between T (temperature) and Φ (neutron flux) when performing iteration .

In this part of the lecture (thermal hydraulics) we limit ourselves only on basic concepts.

We would like to demonstrate how to obtain the *general qualitative thermal hydraulics picture* of a reactor .

For precise thermal hydraulics computation we must have recourse to available programs like

RELAP-5 (see Argonne basic courses on Nuclear safety [1])

II. Mechanisms of heat transfer [1]

We have to consider :

1 / Conduction

2 / Convection

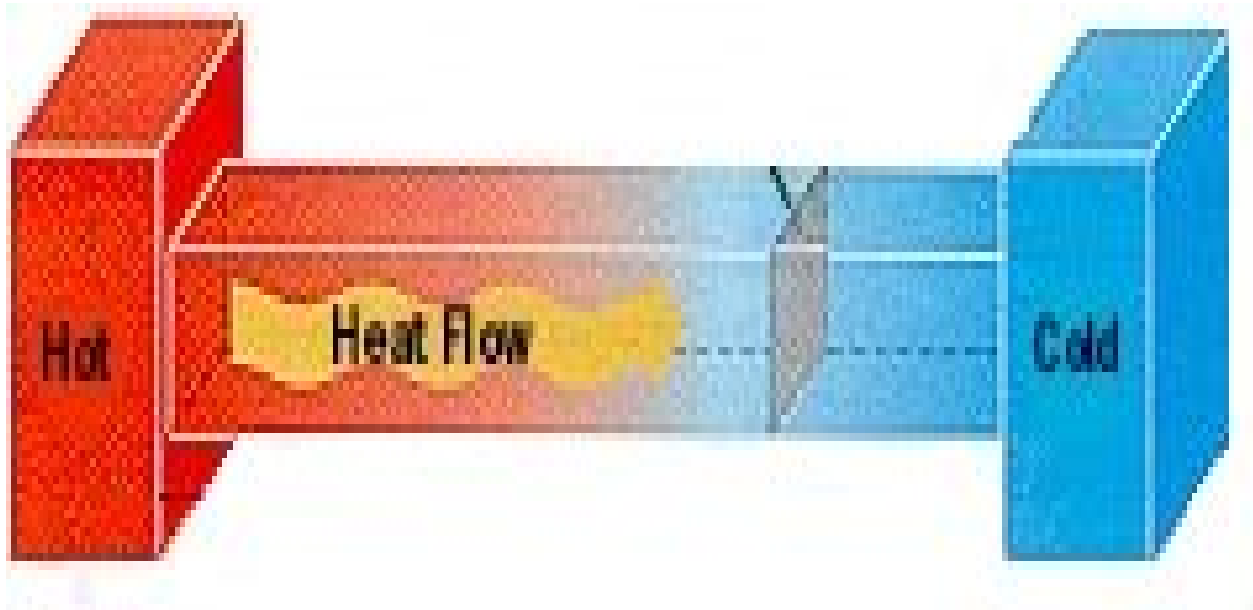
3 / *Radiation*

They are three mechanisms
of heat transfer

Conduction

This heat transfer mechanism takes place in a body, or in bodies in contact, when a temperature difference exists between two regions . Heat flows from the region of high temperatures to the region of low temperatures.

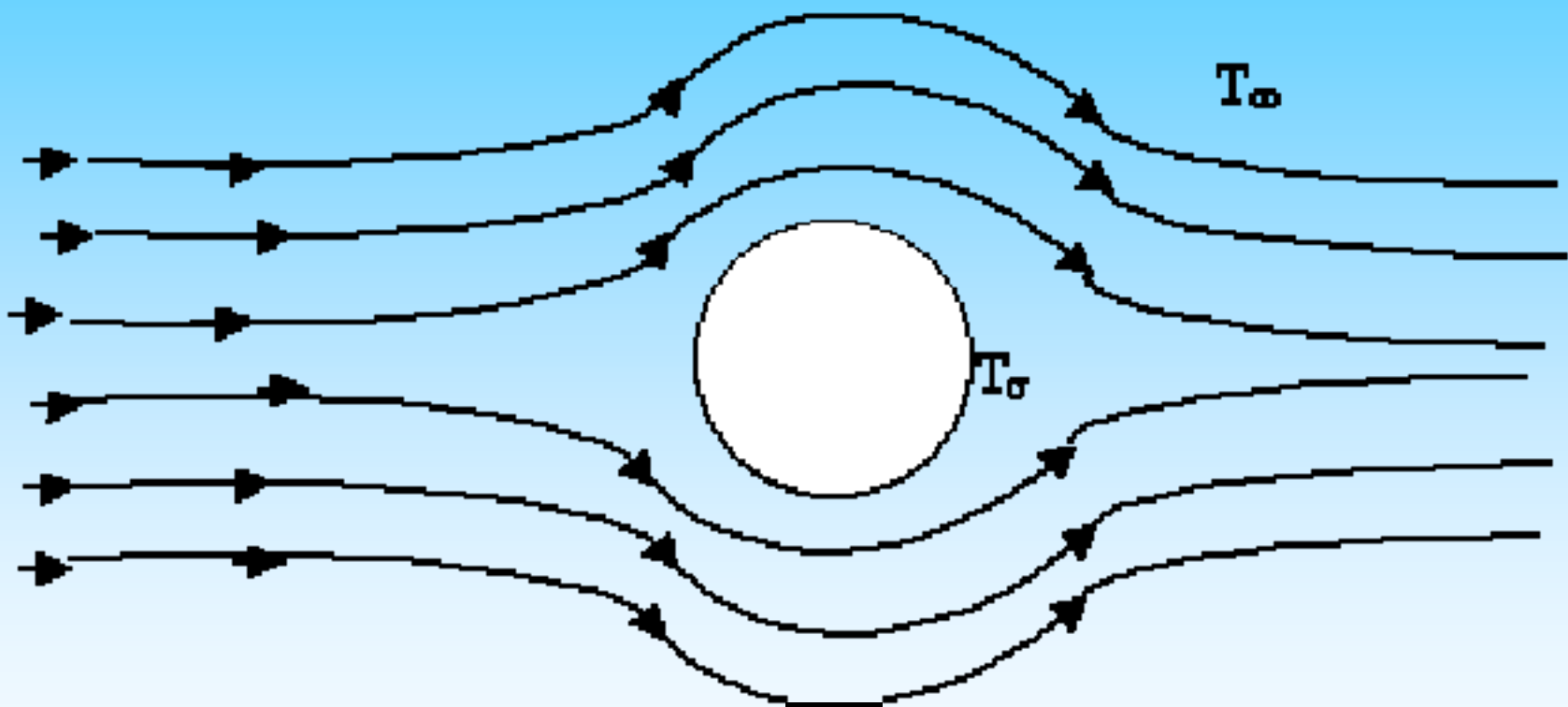
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Convection

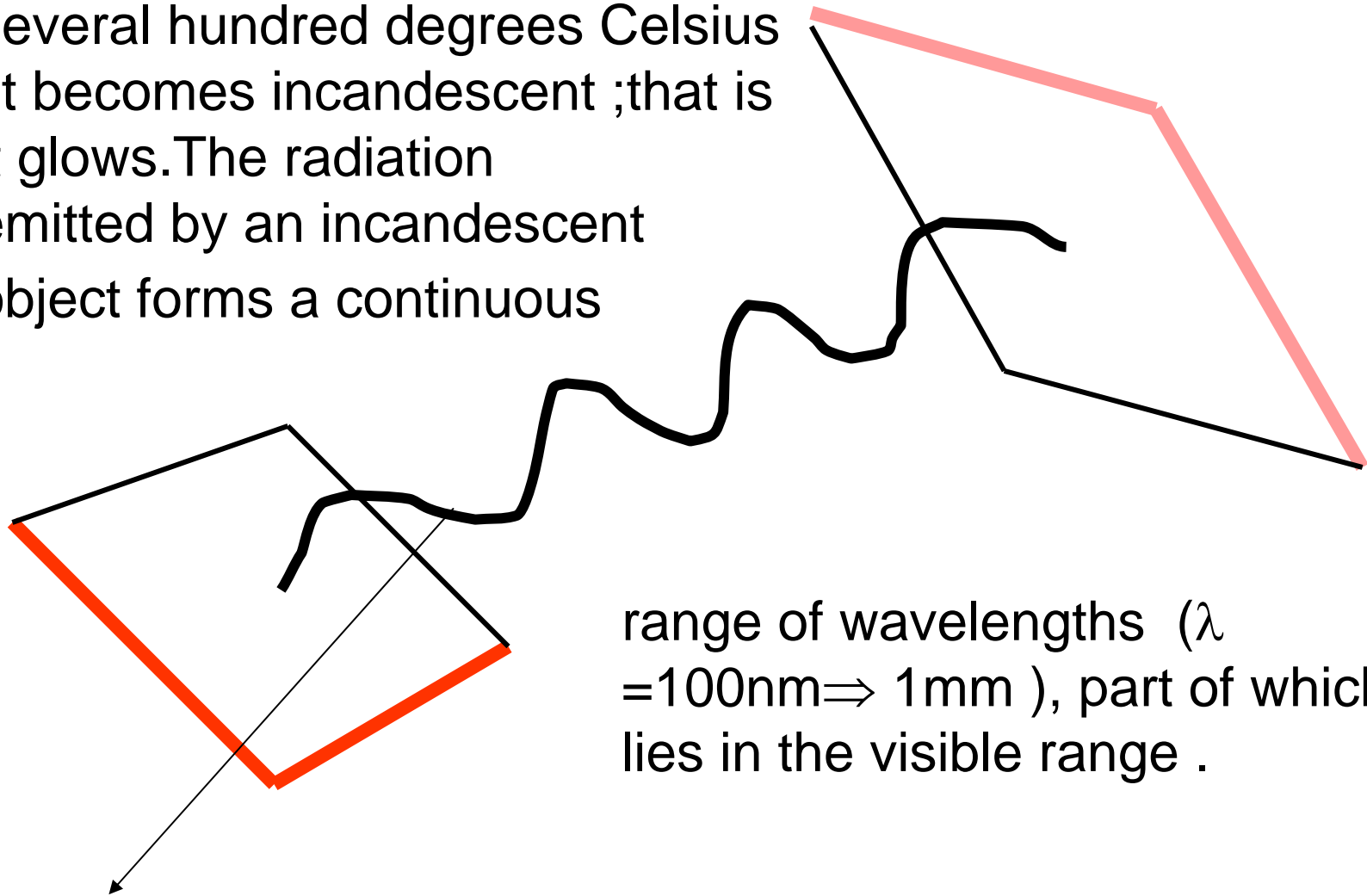
The heat exchanged at the interface between a solid and a flowing fluid is called convection. For example T_σ may be a fuel rod and T_∞ coolant flow.



Radiation

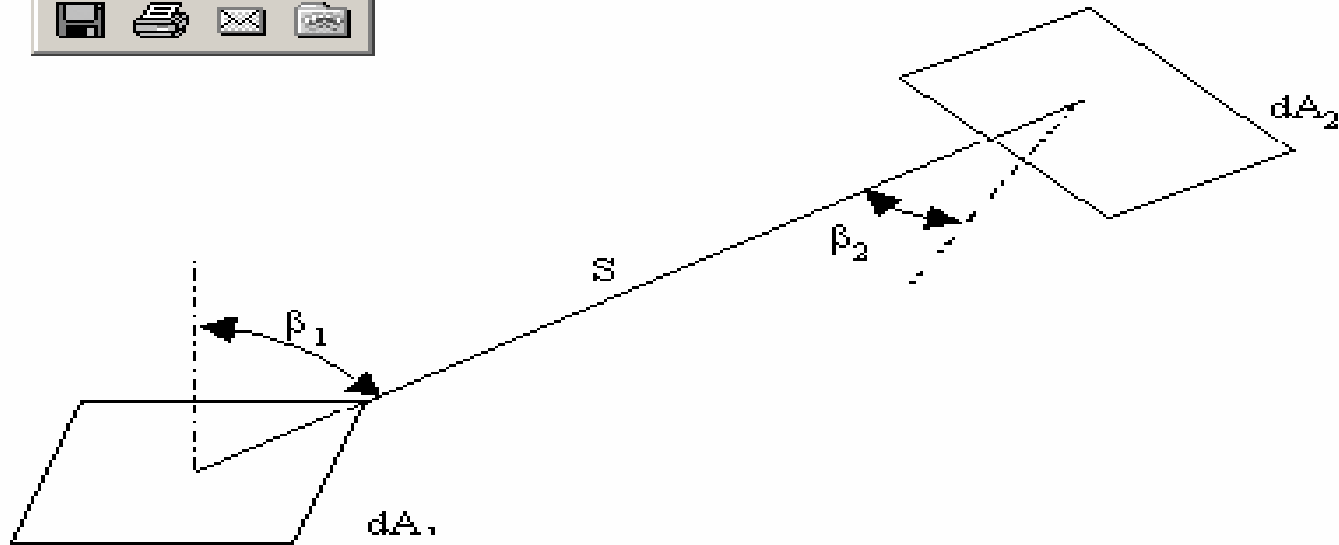
When two bodies (not in contact), at different temperatures, are located in positions that they can “see each other”, a process of heat transfer by [radiation](#) occurs. This type of thermal exchange doesn't require a physical medium (such as air or water) between the bodies and therefore this is the predominant type of heat transfer mode in space.

When a solid object is heated to several hundred degrees Celsius, it becomes incandescent; that is, it glows. The radiation emitted by an incandescent object forms a continuous



range of wavelengths ($\lambda = 100\text{nm} \Rightarrow 1\text{mm}$), part of which lies in the visible range.

Electromagnetic wave



For illustration consider the case of black body ,here we have [1] :

$$dQ = (\cos \beta_1 \cos \beta_2 / s^2) dA_1 dA_2 \sigma / \pi (T_1^4 - T_2^4)$$

for the heat exchanged between two elementary surfaces. To obtain the expression of the heat balance between two non-elementary surfaces it is necessary to [integrate this equation](#) .

In this equation

σ = Stefan-Boltzmann constant

III. Heat generated and power of the reactor [2]

The heat generated in the fuel is equal to:

$$Q_{\text{total}} = \varepsilon \Sigma_f \Phi V_f \text{ cal /sec} \dots\dots\dots (1)$$

Where $\varepsilon \approx 0.9E$ ($E=200\text{Mev}$)

$[\Sigma_f \Phi]$ = number of fissions /volume.sec

$[\Sigma_f]$ = 1/cm

$[\Phi]$ = number of neutrons /cm².sec

cal =4.1868 joules \Rightarrow [Watt.sec]

The equation (1) indicates the relation between reactor physics (Φ) and thermal hydraulics (Q)

W = power of the reactor =

Q_{total} + other radiations

$$= \sum_f \Phi V_f E$$

$$= 3.2 \times 10^{-14} \sum_f \Phi V_f \text{ kW} \dots\dots\dots (1a)$$

So we have

$$Q_{\text{total}} \approx 0.9 \text{ W}$$

Remember that

$$\Phi = \int \Phi(\mathbf{r}, E) d\mathbf{r} dE / V_f$$

Exercise:

Using the formula (1a) and the fuel configuration of Dalat reactor ,estimate the power of Dalat reactor.

We have here 2 regions : $v_1=69.4 \text{ liter} \times 7/101$ & $v_2 =69.4 \text{ liter} \times 94/101$.

In v_1 :

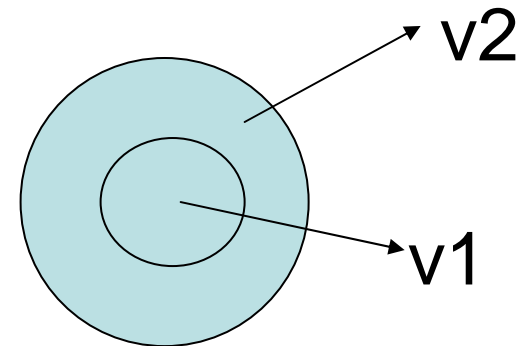
$$\Phi_{\text{fast}} = 5.2 \times 10^{12} \text{ [1/cm}^2 \cdot \text{sec]},$$

$$\Phi_{\text{thermal}} = 4.2 \times 10^{12} \text{ [1/cm}^2 \cdot \text{sec]}$$

In v_2 :

$$\Phi_{\text{fast}} = 3.8 \times 10^{12} \text{ [1/cm}^2 \cdot \text{sec]},$$

$$\Phi_{\text{thermal}} = 3.7 \times 10^{12} \text{ [1/cm}^2 \cdot \text{sec]}.$$



IV . Correlations between temperatures [1] & [2]

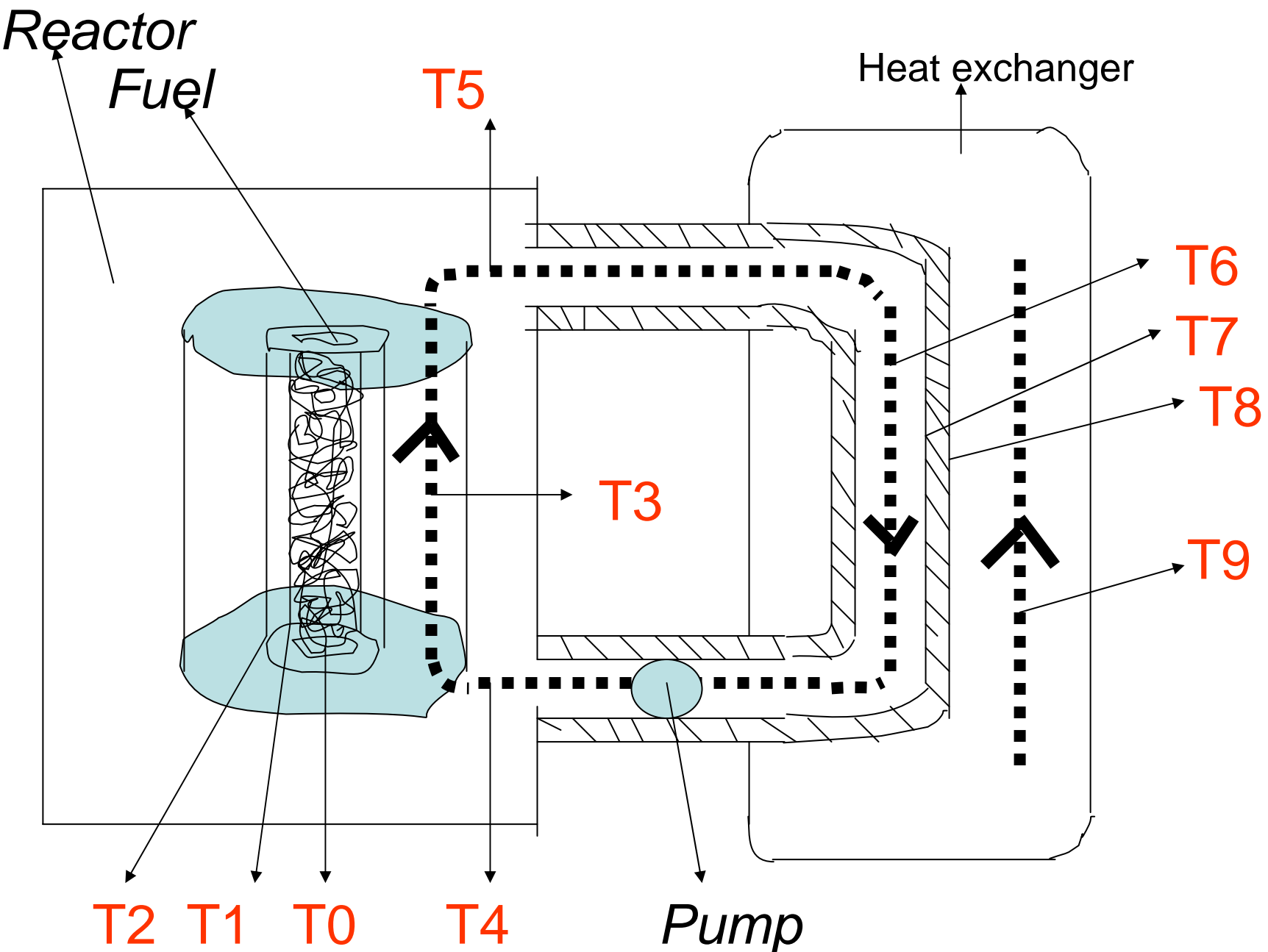
We consider here the following temperatures :

T0 , T1, T2 in the fuel

T3 , T4 , T5 of the part of the loop I , located in
the reactor

T6 , T7 , T8 of the part of the loop I , located in
the steam generator

T9 of the loop II



Let us calculate the temperatures $T(\mathbf{r}, t)$:.

A / Consider first the conduction mechanism

(T_0, T_1, T_2)

We have the equation (Fourier's Law) :

$$\mathbf{q} = -k \nabla T \dots \dots \dots (2)$$

Where k =thermal conductivity =[cal/cm.sec.°C]

q = (density of) heat flux = [cal / cm².sec]

Because the heat confined in a element of volume is equal

to: $c_p \rho dv.T$

where c_p = specific heat at constant pressure= [cal/g.°C]

We have the following equation:

$$\frac{\partial}{\partial t} \int c_p \rho dV \cdot T =$$

$$k \int \nabla T \cdot df \text{ (df=surface element)} \dots \dots \dots - \text{flux}$$

$$+ Q_{\text{total}} \dots \dots \dots \text{source}$$

Now using Gauss theorem ($\int \dots dV \Rightarrow \int \dots df$) we have

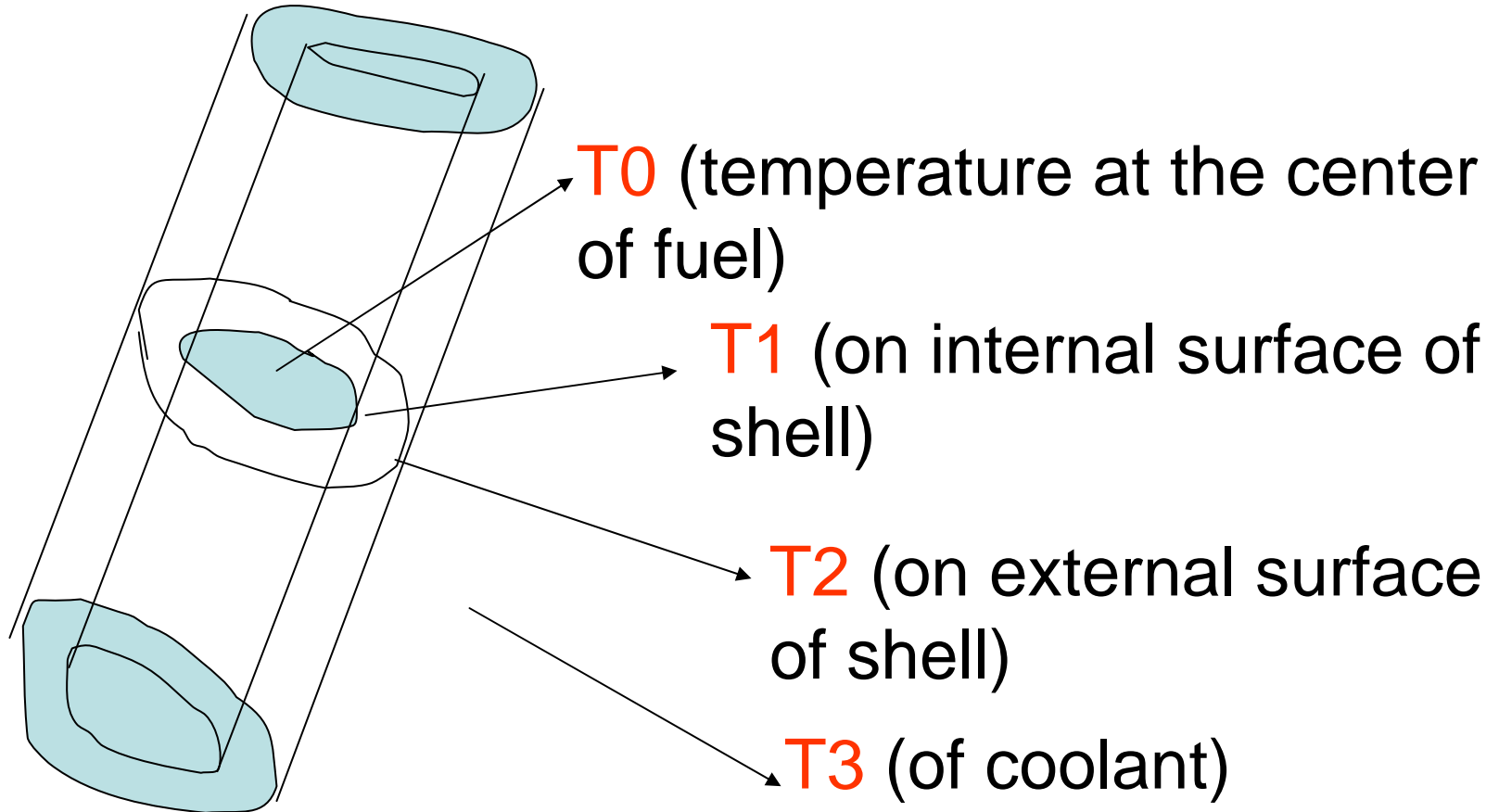
$$c_p \rho \frac{\partial T}{\partial t} = k \nabla T + Q(\mathbf{r}, t) \dots \dots \dots (3)$$

By solving this equation we can find the temperature T .

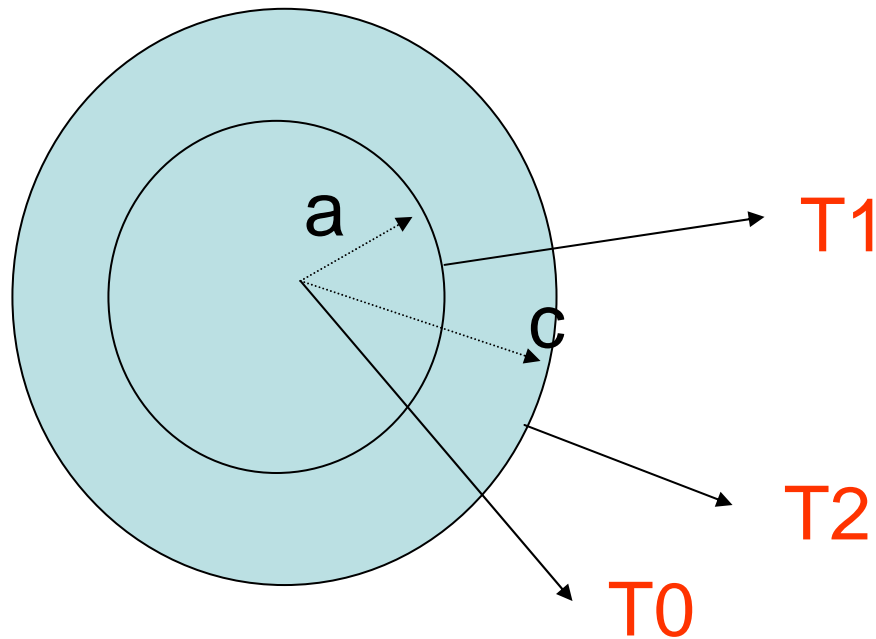
In the stationary case we have :

$$k \Delta T + Q = 0 \quad (\text{steady-state heat conduction equation})$$

Consider a fuel element



We have here a fuel element with different temperatures at different surfaces.



a = radius of fuel

$c - a$ = thickness of the shell (cladding)

Suppose that the length l of the fuel satisfies the condition :

$$l \gg a$$

So T will not depend on z and

$T = T(r, \varphi, t)$ in fact by polar symmetry $\Rightarrow T(r)$

And we can use the cylindrical coordinates and obtain :

$$k \left(\frac{1}{r} \right) \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = -Q \dots\dots\dots(4)$$

Denote T_{fuel} by T_f and T_{shell} by T_s we have two equations valid in the fuel and in the shell :

$$T''_f + 1/r T'_f = -Q/k \dots\dots\dots (5)$$

$$T''_s + 1/r T'_s = 0 \text{ (no source in the shell)} \dots\dots\dots (6)$$

Boundary conditions :

When $r=0$

$$T(0) = T_0$$

When $r = a$

$$T_f(a) = T_s(a) = T_1, \quad [k_f \nabla T_f = k_s \nabla T_s]_{r=a}$$

When $r = c$

$$T_s(c) = T_2 \neq T_3$$

Using the boundary conditions and solving (5)&(6),
we have :

$$T1 = T0 - Qa^2 / 4k_f$$

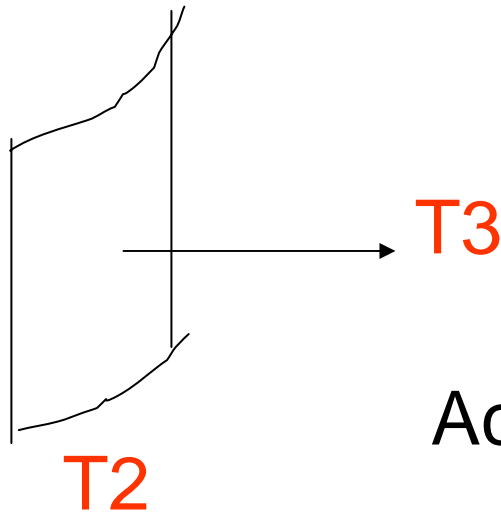
$$T2 = T1 - (Qa^2 / 2k_s) \ln c/a$$

So if T0 is known we can determine T1 & T2.

T0 can be calculated if T3 is known (see (8)).

B / *Consider now the convection mechanism .*

(**T2 , T3**) \Rightarrow heat transfer to coolant

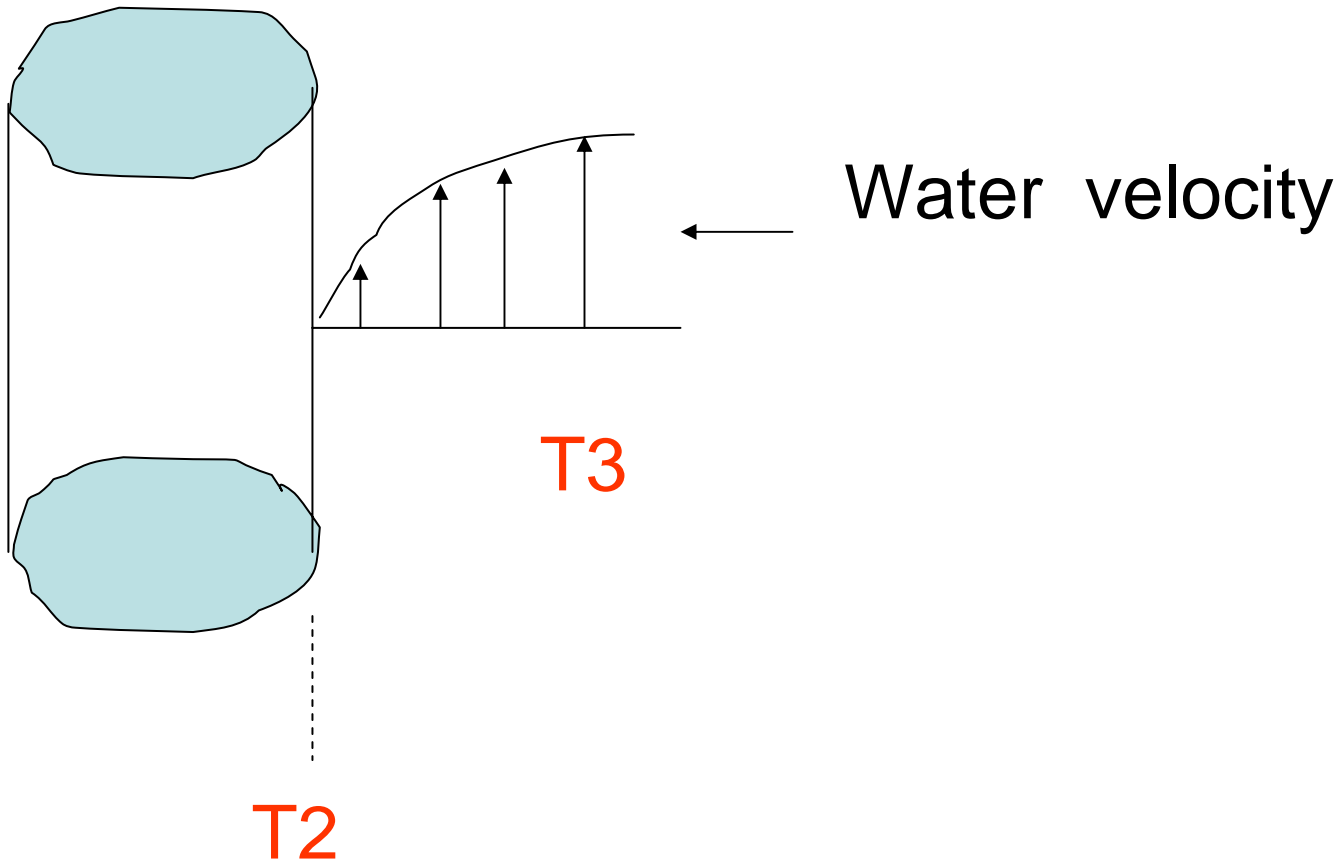


According to the Newton law :

$$q_{(r=c)} = \alpha (T2 - T3) \dots\dots(7)$$

Where α = heat transfer coefficient

with dimension [cal /cm².sec.⁰C]



When $r = c$ we have $\alpha (T_2 - T_3) = -k_s (\nabla T_s)_{(r=c)}$

After some calculations , we obtain :

$$T_0 = T_3 + \frac{Qa^2}{2c\alpha} + \left(\frac{Qa^2}{2k_s}\right) \frac{\ln c/a}{a} + \frac{Qa^2}{4k_f} \dots \dots \dots (8)$$

So we have a relation between T0 & T3 .

Remark: When the Reynolds number > 4000 , we have a turbulent motion of the fluid .

Reynolds number $Re = Dv\rho / \eta$

D=hydraulics diameter

V= velocity

ρ = density , η =viscosity

In the case of natural convection ,the coefficient

α can be calculated by using the numbers Grashof G and Prandtl P .

$$\alpha = 0,52 \text{ k/d } [G \cdot \Delta T]^{0.25} P^{0.25}$$

The temperature figures in the following equation :

Navier-Stokes:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} = -\nabla p / \rho + \nu \Delta \mathbf{v} + \mathbf{g} \dots \dots \dots (9)$$

:

Heat transfer equation

$$c_p \rho (\partial T / \partial t + \mathbf{v} \Delta T) = k \Delta T + Q \dots \quad (10)$$

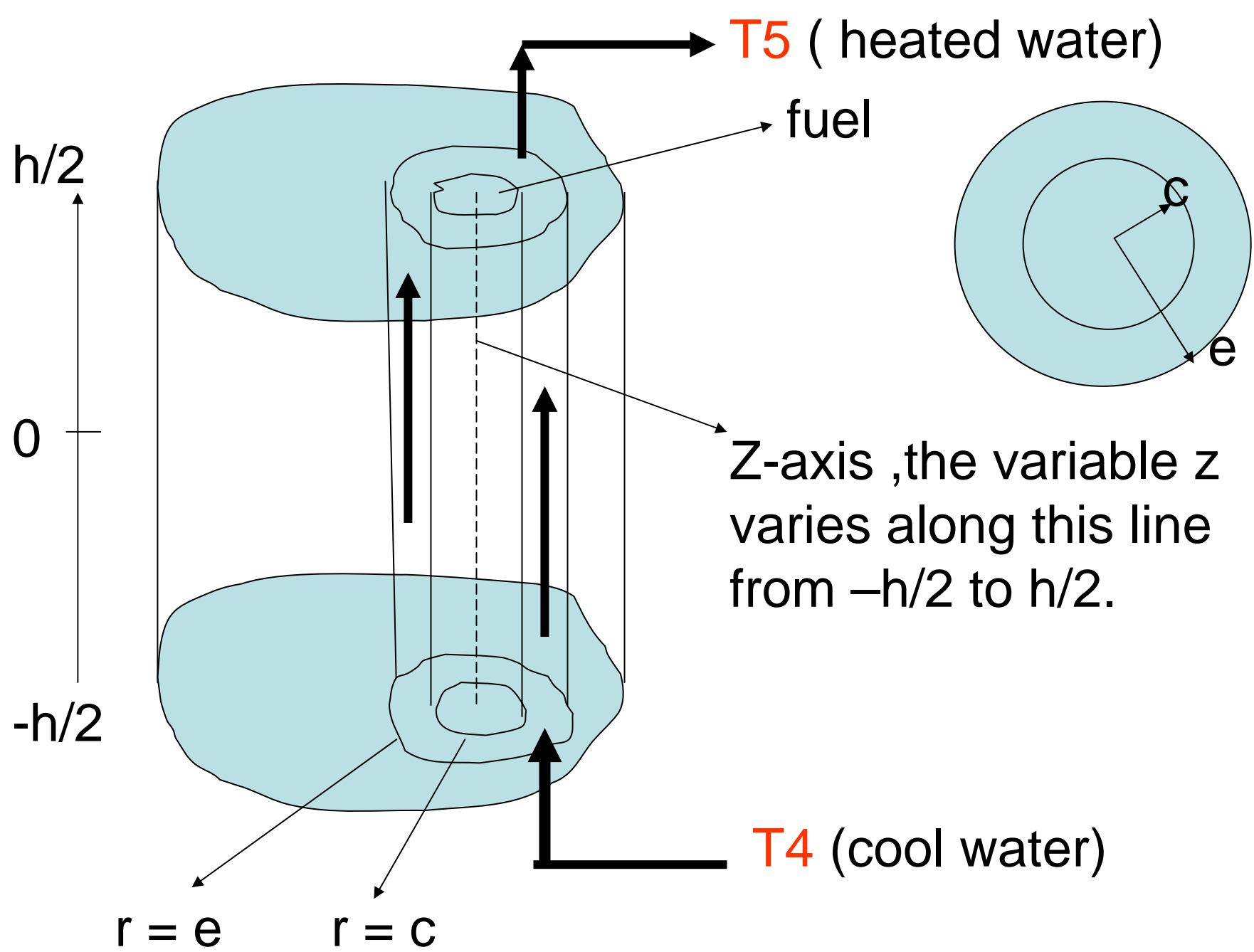
Continuity equation :

$$\text{div } \mathbf{v} = 0 \dots \dots \dots (11)$$

Here we have 5 equations for 5 variables T , \mathbf{v} , p/ρ .

C . Consider now the dependence on z in the fuel problem

(T4, T5) \Rightarrow temperature along a coolant channel



Let $B = \pi (e^2 - c^2)$ = cross surface of coolant around the fuel. After some calculations we obtain:

$$T_3(z) = T_4 - [\pi a^2 / (\rho v B c_p)] \int Q(z') dz' \dots\dots\dots (12)$$

(\int over $-h/2 \Rightarrow z$, $T_4 = T_3_{z = -h/2}$).

Now suppose that $Q(z)$ has the following form:

$$Q(z) = \frac{1}{2} Q_{\max} \cos(\pi z/h)$$

we can obtain the following relation :

$$T_5 = T_4 + V_f \Sigma_f \varphi_{\max} \varepsilon / (\rho v c_p B \pi) \dots\dots\dots (13)$$

Where $\varphi_{\max} = (\int \Phi(r, E) dE)_{\max}$ with respect to r

$$(T5 = T3_{z=+ h/2})$$

From here we can determine the point z_{\max} ,
where $T2 \Rightarrow T2_{\max}$ and

$$T0_{\max} = T2_{\max} + [a^2/4k_f + (a^2/2k_s) \cdot \ln c/a] Q_{z=z_{\max}}$$

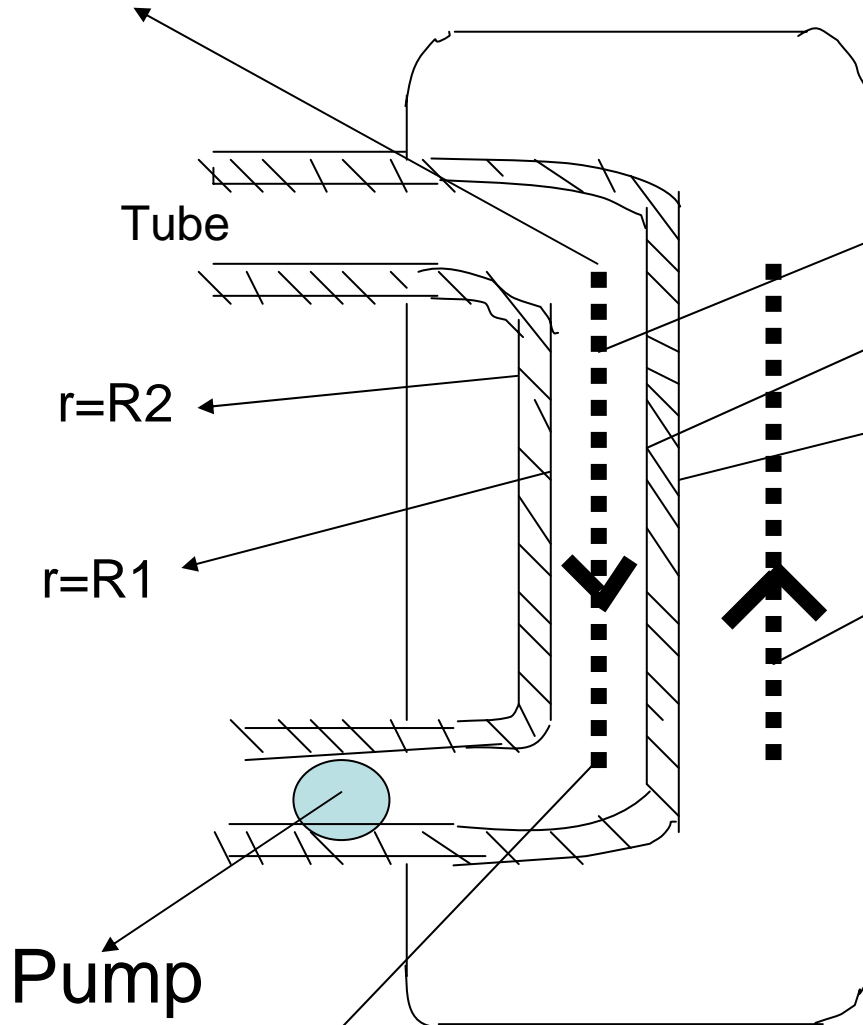
So the fuel experiences a temperature tension =

$$T0_{\max} - T2_{\max}$$

D . Consider now the heat exchange between the loops I & II

$$(T6 , T7 , T8 , T9)$$

$X = 0, T6 = T5$



$T6(x)$ water in loop I

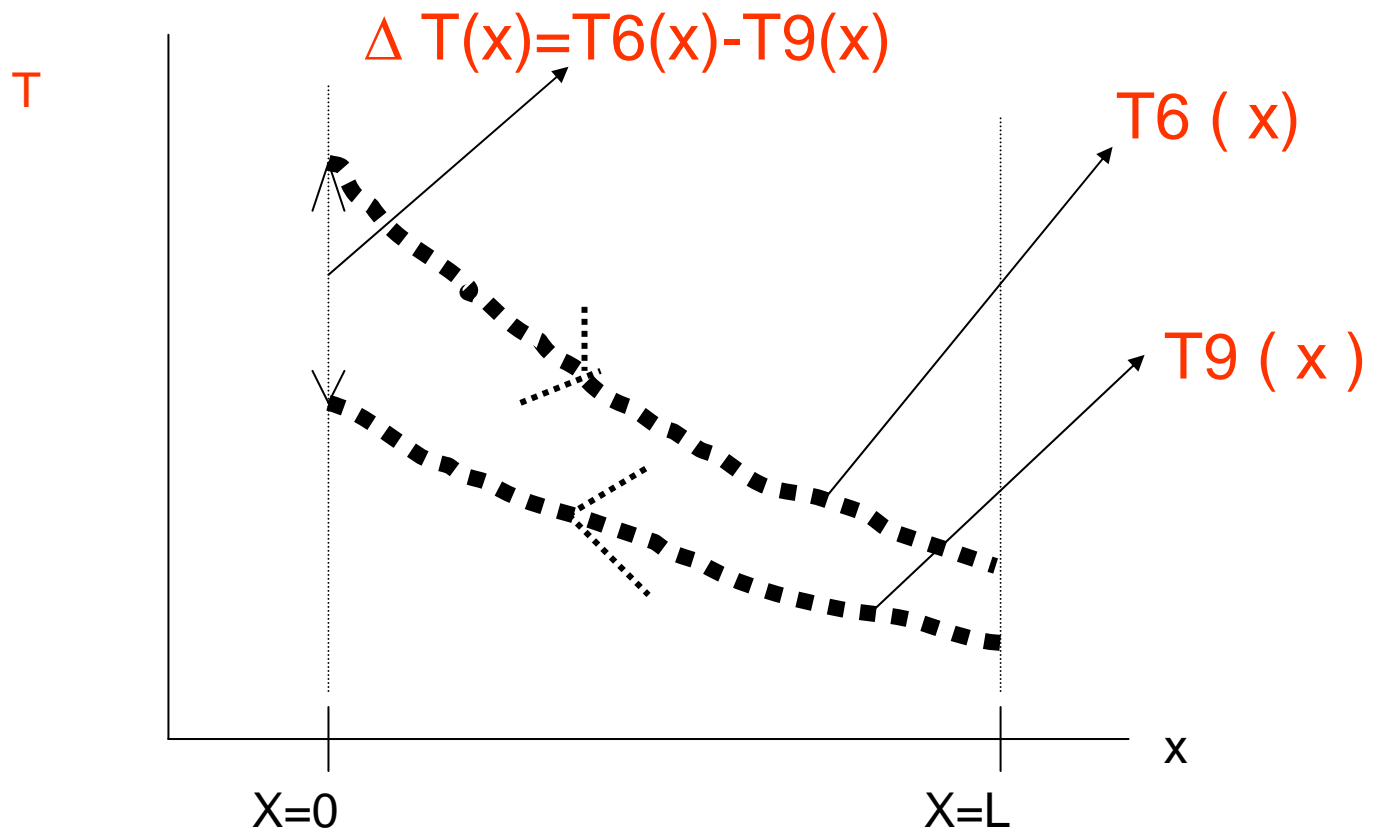
$T7(x)$

$T8(x)$

$T9(x)$ water in loop II, $T9$ decreases when x increases (counter-current flow)

The variable x varies along these dashed lines from 0 to L (L =length of the tube, conducting the coolant I in the heat exchanger).

$X = L, T6 = T4$



Counter-current flow [1], Thermal hydraulics .

The point $x=0$ corresponds to the exit of the reactor

$$\Leftarrow T6_{x=0} = T5$$

The point $x=L$ corresponds to the entrance of the reactor

$$\Rightarrow T6_{x=L} = T4$$

Denote $R1$ & $R2$ the radii of the tube ($R1 < R2$)

$$T_{(r=R1)} = T7(x)$$

$$T_{(r=R2)} = T8(x)$$

We can determine the temperature $T9_{x=0}$, this temperature is the working temperature (temperature of the water vapor at the entrance of turbine) .

$$T9_{(x=0)} = T4 + \alpha_2 [T8_{(x=0)} - T9_{(x=0)}] \times$$

$$\times [(\frac{R^2}{k}) \ln(\frac{R2}{R1}) + \frac{1}{\alpha_2} + \frac{R2}{R1 \alpha_1}]$$

(14)

α_2 & α_1 = heat transfer coefficients at $r=R2$ & $R1$ respectively

If the water of the loop II circulates with the velocity v_{II} , then the work A provided is equal to

$$A = 0.116 \cdot 10^{-5} \eta \rho_{II} v_{II} B_{II} c_{p,II} [T9_{(x=0)} - T9_{(x=L)}] t \text{ kWh}$$

where η =efficiency, ρ_{II} = density, B_{II} = cross section of the water flow of loop II, $c_{p,II}$ =specific heat, t =time

We can also obtain the following formula :

$$Q_t = \rho v B c_p (T5 - T4) \dots \dots \dots (14a).$$

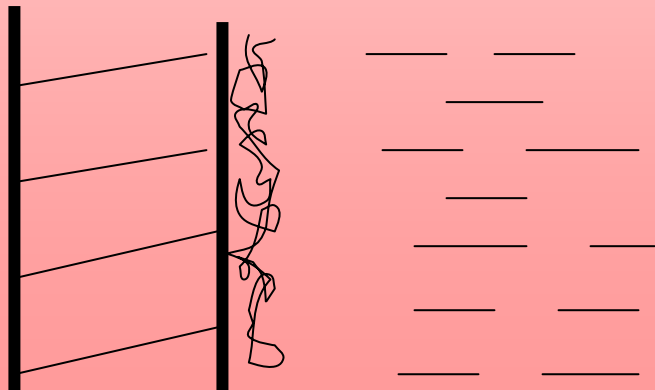
Exercise :

Using (14a) estimate v (loop I) for the Dalat reactor,
with $B = 0,055 \text{ m}^2$ and $\Delta T \approx 20 \text{ }^\circ\text{C}$.

Remark : If taking account of steam generation , the formulae in paragraph *D* become complicated
(steam generation heat must be involved)

V . Phases in the coolant [2]

The boiling with the appearance of vapor bubbles is acceptable for the beginning when the number of bubbles is not great ,because these bubbles mix the liquid phase and increase the contact of liquid phase with the fuel. But when the number of bubbles becomes very great , they form a sort of vapor membrane that prevents the contact of coolant with fuel $\Rightarrow \alpha$ will decrease .This situation can lead to accidents.



Boiling regimes [1] & [3]

1 . As the surface temperature of the fuel is increased further , a point is eventually reached where bubbles of vapor begin to form at various imperfections on the surface of the fuel rod \Rightarrow ***nucleate boiling***

nucleate boiling (local boiling \rightarrow bulk boiling)

The formation of more and more vapor close to the surface of the heater element has the effect of gradually insulating the surface. The point where the heat flux is maximum (of the order of 10^6 W/m²) is denominated "*Burnout point*" and the heat flux "*Critical Heat Flux*".

2 . Boiling crisis

When the bubble density becomes so great that adjacent bubbles coalesce and begin to form a vapor film across the surface of the rods ,the system is said to be in a boiling crisis

Film Boiling [1]

From the point the vapor film recovering the surface of the heater surface becomes stable and the *Film Boiling* regime is established. The radiation through the vapor film plays an important role as the temperature difference increases. This regime persists until the temperature of the heater surface reaches the melting point of the material.

At a certain height, the vapor film is completely evaporated (Dryout Point). For systems controlled by the heat fluxes, the wall temperature rises quickly. Although liquid drops still exist in the flowing mixture, the heat transfer regime is known as "Liquid Deficient Region". In the event that the whole liquid is evaporated a single-phase flow of vapor occurs. This is a significant issue for a reactor. This type of situation and other forms of a loss of cooling have been the cause of numerous accidents.

VI . Pump [2]

How to calculate the power of a pump?

The power of a pump depends on the loss of the pressure along the loop .

The loss of pressure is due to :

- *Resistance of the internal surface of the shell*
- *Variations of velocity (the cross section varies , the direction of the flux varies ,...)*
- *Boiling events*
- *Different heights to overcome*
- ...

We have the following formula:

$$\Delta p = -\rho g \Delta h - \rho/2 \sum v_i^2 K_i - \Delta p_{\text{formation of vapor}}$$

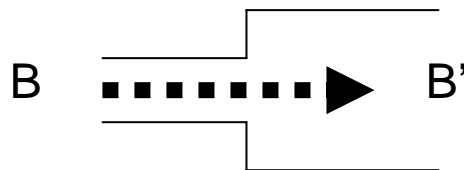
..... (15)

where K_i = empirical constants (available)

For example :

$$K_{\text{fiction}} = \mu (4L/d)$$

$$K_{\text{section enlargement}} = (1 - B/B')^2, \text{ B' enlargement of B}$$



In the formula (15) Σ = sum over i (i = different causes of pressure loss at different points along the circuit)

The power of the pump is then :

$$W_{\text{pump}} \text{ [kW]} = 10^{-10} \Delta p B v n \dots\dots\dots(16)$$

where n = number of channels

B= cross section of channels .

VII . Conclusion

So we have established the correlations between the temperatures at different important points in the loops I & II .

These temperature values are necessary as mentioned above for the feedback calculation of Φ (neutron flux) .

When the new Φ will be calculated we must perform again thermal hydraulics calculation and so on, until this iteration procedure reaches the wanted precision . For this purpose we must have recourse to available codes [1]).



Thank you